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Suppressed supersymmetry breaking terms in the Higgs sector

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Abstract

We study the little hierarchy between mass parameters in the Higgs sector and other SUSY breaking masses. This type of spectrum can relieve the fine-tuning problem in the MSSM Higgs sector. Our scenario can be realized by superconformal dynamics. The spectrum in our scenario has significant implications in other phenomenological aspects like the relic abundance of the lightest neutralino and relaxation of the unbounded-from-below constraints.

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1 Introduction

Supersymmetric extensions of the Standard model is postulated as a solution for the hierarchy problem. The radiative electroweak symmetry breaking is one of the attractive aspects of the supersymmetric model: the large top Yukawa coupling derives the Higgs mass to be tachyonic in the low energy [1]. Within the framework of the minimal supersymmetric standard model (MSSM), the theoretical upper bound of the lightest Higgs boson mass m_{h^0} at the tree-level is equal to the Z boson mass M_Z. On the other hand, the current experimental lower bound is 115 GeV. A large correction to the Higgs mass can be obtained when the stop mass is as large as 500 GeV or more. Such large stop mass squared induces the negative and comparable size of SUSY breaking Higgs scalar mass squared through the renormalization group (RG) effect. That requires fine-tuning to lead to the weak scale from SUSY breaking parameters of the magnitude of 500 GeV or higher mass scale.

So far, several mechanisms to solve the fine-tuning problem have been proposed [2, 3, 4, 5, 6]. One way is to assume additional quartic couplings of the Higgs fields other than $SU(2) \times U(1)_Y$ D-terms by extension of the gauge sector, the Higgs sector and so on. Another is to increase the theoretical prediction of the lightest Higgs scalar mass at tree level, which may be achieved also by the additional quartic couplings, and to make the radiative correction to Higgs mass to be small. In these mechanisms most soft scalar masses and μ-parameter are often assumed to be relatively small to avoid the fine-tuning.

In this paper, we consider the little hierarchy between SUSY breaking parameters in the Higgs sector and others. Then, we propose that such suppressed magnitude of mass parameters only in the Higgs sector, SUSY breaking Higgs masses and also the μ-parameter, improves the fine-tuning problem. Such little hierarchy can be realized e.g. by superconformal dynamics, which must be decoupled around $O(1) - O(10)$ TeV. Furthermore, the little hierarchy between SUSY breaking parameters lead to the spectrum different from the usual one, e.g. the minimal supergravity model. Since the μ-term is small, the lightest neutralino is not purely bino-like. Also the constraint about charge/color breaking and unbounded-from-below directions can be ameliorated.

This paper is organized as follows. In Section 2, we explain the fine-tuning problem in the MSSM, and we show our scenario to relax the problem. We propose the little hierarchy between the mass parameters in the Higgs sectors and other SUSY breaking masses like the stop mass. In section 3, we will show certain models to realize such little hierarchy by the superconformal dynamics. In section 4, we will discuss generic phenomenological aspects of our scenario. Section 5 is devoted to conclusion and discussions.

2 Suppressed supersymmetry breaking in Higgs sector

The Higgs sector of the MSSM has the following potential for the neutral components, $H^0_{u,d}$,

$$\begin{aligned} V(H^0_u, H^0_d) &= m_1^2|H^0_d|^2 + m_2^2|H^0_u|^2 - 2\mu B H^0_u H^0_d \\ &+ \frac{1}{8}(g^2+g'^2)(|H^0_d|^2 - |H^0_u|^2)^2, \end{aligned} \tag{1}$$

with $m_1^2 = m^2_{H_d} + \mu^2$ and $m_2^2 = m^2_{H_u} + \mu^2$, where $m^2_{H_d}$, $m^2_{H_u}$ and μB are SUSY breaking parameters, while μ is the supersymmetric mass parameter in the superpotential $\mu H^0_u H^0_d$. The last term comes from the $SU(2) \times U(1)_Y$ D-terms.

If the following two conditions:

$$m_1^2 m_2^2 < (\mu B)^2, \tag{2}$$

$$m_1^2 + m_2^2 > 2|\mu B|, \tag{3}$$

are satisfied, the electroweak symmetry is broken. The latter is the condition to avoid the direction of the potential unbounded from below along $\langle H^0_u\rangle = \langle H^0_d\rangle$. Then the vacuum expectation value v, where $v^2 = \langle H^0_u\rangle^2 + \langle H^0_d\rangle^2$, is obtained by Higgs mass parameters as

$$v^2 = \frac{4}{g^2+g'^2}\left(-\mu^2 + \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1}\right), \tag{4}$$

where $4/(g^2+g'^2) \approx 7$ at M_Z, and $\tan\beta \equiv \langle H^0_u\rangle/\langle H^0_d\rangle$. For a moderate value of $\tan\beta$, it reduces to

$$v^2 \approx \frac{4}{g^2+g'^2}(-\mu^2 - m^2_{H_u}). \tag{5}$$

Its magnitude v itself should be derived as $v = 174$ GeV. The natural scale of μ^2 and $m^2_{H_u}$ is $O(M_Z^2)$.

The theoretical prediction for the lightest CP-even Higgs boson mass m_{h^0} at the tree level is equal to the Z boson mass M_Z or less. The dominant one-loop correction is due to the top-stop mass splitting, because the top Yukawa is large. Thus, the lightest Higgs mass m_{h^0} at the one-loop is written as [7]

$$m^2_{h^0} \le M_Z^2 + \frac{3}{4\pi^2} Y_t^4 v^2 \ln\left(\frac{m^2_{\tilde{t}}}{m_t^2}\right), \tag{6}$$

where Y_t is the top Yukawa coupling, and m_t and $m_{\tilde{t}}$ are the top and stop masses, respectively. The current experimental lower bound of m_{h^0} is equal to 115 GeV. That requires $m_{\tilde{t}} \ge 500$ GeV through Eq. (6).

On the other hand, the SUSY breaking scalar mass m_{H_u} receives the one-loop radiative correction between the high energy scale Λ and the weak scale,

$$\Delta m_{H_u}^2 \sim -12 \frac{Y_t^2}{16\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda}{v}. \tag{7}$$

When we take e.g. $\Lambda = 10^{16}$ or 10^{18} GeV, we evaluate

$$\Delta m_{H_u}^2 \sim -m_{\tilde{t}}^2. \tag{8}$$

This rather large radiative correction is due to the long running scale, i.e. $\ln \frac{\Lambda}{v} \sim 30$. Hence, the natural order of $|m_{H_u}^2|$ at the weak scale is equal to $O(m_{\tilde{t}}^2)$ when the initial value of $m_{H_u}^2$ at Λ is comparable with $m_{\tilde{t}}^2$ or less.

Appearance of the same order soft scalar masses, $m_{H_u}^2$ and $m_{\tilde{t}}^2$, at low energy may be also explained in terms of the fixed point behavior. The MSSM has the Pendleton-Ross fixed point [8] for the top Yukawa coupling. That is given as

$$Y_t^2 = (4\pi) \frac{7}{18} \alpha_3, \tag{9}$$

when we neglect the other gauge and Yukawa couplings. At this fixed point, the SUSY breaking terms also satisfy the specific relation,

$$m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + m_{H_u}^2 = M_3^2. \tag{10}$$

Thus, around this fixed point, we have

$$m_{H_u}^2 \approx -m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 + M_3^2. \tag{11}$$

Note that the stop mass squared has the radiative correction $\Delta m_{\tilde{t}}^2$ between Λ and the weak scale is of $O(M_3^2)$ e.g. for $\Lambda = 10^{16} - 10^{18}$ GeV. Again, the natural order of $|m_{H_u}^2|$ is of $O(m_{\tilde{t}}^2)$ or more.

Thus, the natural order of $|m_{H_u}^2|$ seems of $O(m_{\tilde{t}}^2)$ or more, and the lower mass bound of the lightest Higgs scalar field, $m_{h^0} \geq 115$ GeV, requires that $m_{\tilde{t}} \geq 500$ GeV. That implies that the natural order of $|m_{H_u}^2|$ is of $O(500^2)$ (GeV)^2 or more. However, Eq. (5) requires that μ^2 is also the same order and these two terms should be fine-tuned to lead to the weak scale. Such a fine-tuning is often presented in terms of the following fine-tuning parameter [9, 10],

$$\Delta_{a^2} = \left| \frac{a^2}{v^2} \frac{\partial v^2}{\partial a^2} \right|. \tag{12}$$

For example, for $\mu^2, |m_{H_u}^2| \sim 500^2$ (GeV)^2, we have

$$\Delta_{\mu^2} = O(100), \qquad \Delta_{m_{H_u}^2} = O(100). \tag{13}$$

Similarly, when $m_{H_u}^2 \sim -m_{\tilde{t}}^2$, we have

$$\Delta_{m_{\tilde{t}}^2} = O(100). \tag{14}$$

These values Δ_{a^2} increase linearly as $\mu^2, |m_{H_u}^2|$ and $m_{\tilde{t}}^2$ increase.

Thus, the combination of Eqs. (5), (6), (8) as well as the experimental lower bound of the Higgs mass leads to unnaturalness. This unnatural situation can be improved if at least one of these relations is modified. Actually, several types of ideas have been proposed. Some of them proposed additional quartic terms of the Higgs fields in the potential like $\delta\lambda_4 |H_u|^4$ [2, 5, 6], e.g. by introducing singlet fields, which have couplings with $H_u H_d$ in the superpotential, or by assuming new D-terms due to extra gauge symmetries, under which the Higgs fields have nontrivial representations. Also, additional quartic terms can be generated by strong dynamics near the weak scale [3] or low scale SUSY breaking [4]. These additional quartic terms change the relation Eq. (5) and, therefore, reduce the fine-tuning parameter directly. In these cases, the relation Eq. (6) is also changed, and the tree level lightest Higgs mass m_{h^0} can be raised up [1]. Then, we do not need a large stop mass. Thus relatively small scalar masses are allowed and the fine-tuning can be ameliorated.

Here, we study the possibility for changing the radiative correction (8) due to the stop mass, and we propose the scenario that SUSY breaking terms in the Higgs sector, e.g. $|m_{H_u}^2|$, are suppressed compared with other SUSY breaking terms through some mechanism. In the next section, we will show a certain model by the superconformal dynamics as an illustrating model to lead to suppressed SUSY breaking in the Higgs sector compared with other SUSY breaking terms. We expect the scenario that $-m_{H_u}^2 \approx 100^2 - 200^2$ (GeV)^2 while sfermion masses are heavier, e.g. $m_{\tilde{t}} \geq 500$ GeV. In this scenario, the μ-parameter must also be suppressed, i.e. $\mu^2 \sim -m_{H_u}^2$. The other SUSY breaking parameters in the Higgs sector, m_{H_d} and B, do not need to be the same order as μ and $O(m_{H_u})$. Thus, we can expect the two cases:

$$(a) \qquad m_{\tilde{t}}^2, m_{H_d}^2 \gg \mu^2 \sim |m_{H_u}^2|, \tag{15}$$

and

$$(b) \qquad m_{\tilde{t}}^2 \gg m_{H_d}^2 \sim \mu^2 \sim |m_{H_u}^2|. \tag{16}$$

Actually, the former case is realized in the next section by use of the superconformal dynamics. The latter can also be realized by the superconformal dynamics, however, alternatively this may be realized e.g. in extra dimensional models, where the whole Higgs sector is separated away from the SUSY breaking source, while others like gaugino and matter fields are near the SUSY breaking source

[1] Additional Higgs mass terms Δm_{h_0} may be generated as hard SUSY breaking terms through strong dynamics or low scale SUSY breaking.

[2]. It is noted that $B^2 \gg \mu^2 \sim |m^2_{H_{u,d}}|$ does not lead to successful electroweak symmetry breaking, since the Higgs potential becomes unbounded from below. Therefore the initial value of B must be properly suppressed in the latter case [3].

In the former case, the relation $B^2 \gg \mu^2$, which is expected in the scenario using superconformal dynamics, is allowed. The value of $\tan\beta$ is obtained as

$$\frac{2\tan\beta}{1+\tan^2\beta} = \frac{2\mu B}{m_1^2 + m_2^2}. \tag{17}$$

In the case with $m^2_{H_d}, B^2 \gg \mu^2 \sim |m^2_{H_u}|$, the value of $\tan\beta$ is obtained as

$$\tan\beta = \frac{m^2_{H_d}}{\mu B}. \tag{18}$$

Thus, we have a moderate or large value of $\tan\beta$, but not a small value like $\tan\beta \sim 1$. There are other phenomenological features in this scenario, and those will be discussed in Section 4 after showing an illustrating model in the next section.

Our primary purpose is to propose the scenario that the Higgs mass parameters are suppressed at the low energy compared with other SUSY breaking parameters. Such a scenario can be realized by the superconformal dynamics. The superconformal sector must be decoupled around $O(1)-O(10)$ TeV. This is because if the decoupling scale is higher, again we have a large radiative correction (8). Such a low decoupling scale of the superconformal sector may induce additional quartic coupling terms in the Higgs potential like $\delta\lambda_4|H_u|^4$ as threshold corrections and/or additional Higgs mass corrections Δm_{h_0}. In general these by-products can also work to relieve unnaturalness of the MSSM.

3 Illustrating model

Now we consider the case that the soft scalar mass of the Higgs fields and also the μ-parameter are suppressed due to strong dynamics, while other soft supersymmetry breaking parameters are remained unsuppressed. To be explicit, let us present certain models, in which an extra superconformal gauge sector strongly coupled with the Higgs fields plays just this role. We introduce an $SU(4)$ gauge symmetry and the following extra matter fields other than the MSSM field content;

$$\begin{array}{lcl lcl} T & : & (\mathbf{4},\mathbf{3},\mathbf{1},-1/3), & \bar{T} & : & (\bar{\mathbf{4}},\bar{\mathbf{3}},\mathbf{1},1/3), \\ D & : & (\mathbf{4},\mathbf{1},\mathbf{2},1/2), & \bar{D} & : & (\bar{\mathbf{4}},\mathbf{1},\mathbf{2},-1/2), \\ S & : & (\mathbf{4},\mathbf{1},\mathbf{1},0), & \bar{S} & : & (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},0), \\ S' & : & (\mathbf{4},\mathbf{1},\mathbf{1},0), & \bar{S}' & : & (\bar{\mathbf{4}},\mathbf{1},\mathbf{1},0), \end{array} \tag{19}$$

[2]The low energy effective theories given by the superconformal dyamics may be realized also in the warped geometry. The related setup has been studied in Ref. [11].

[3]Such a case is allowed if we have additional quartic couplings, e.g. $\delta\lambda_4|H_u|^4$. However the quartic coupling must be large enough to avoid the fine-tuning problem.

where the representations and charges under $SU(4) \times SU(3)_c \times SU(2)_W \times U(1)_Y$ are shown respectively in the brace. Unification of the MSSM gauge couplings in the perturbative regime is not destroyed by these extra matter fields. [4] Also the model is vector-like and, therefore, free from anomalies.

For the moment, we consider this model with neglecting the MSSM gauge and Yukawa interactions. Then the $SU(4)$ ($N_c = 4$) gauge theory contains the flavor number $N_f = 7$ of vector-like matter fields with (anti-)fundamental representation. Therefore this theory belongs to the so-called superconformal window, which is given by $(3/2)N_c < N_f < 3N_c$ for $SU(N_c)$ SQCD [12]. The $SU(4)$ gauge coupling is attracted into a fixed point at low energy and there the extra matter fields $\Phi = (T, D, S, S')$ acquire a large negative anomalous dimension $\gamma_\Phi = -5/7 < -1/2$.

We also assume that the extra matter fields carry odd matter parity (R-parity) just like quarks and leptons. Then allowed Yukawa couplings among extra matter fields and the MSSM Higgs fields are strongly relevant because of the large anomalous dimensions of the extra matter fields. It is well expected that there is another infrared fixed point where the Higgs fields are strongly coupled with the superconformal extra matter fields through Yukawa interactions. Hereafter we assume that the Yukawa couplings become large and the gauge theory approaches into this fixed point around the energy scale $\Lambda_{\rm SC}$. It is noted also that there are other relevant operators than the Yukawa terms because of large anomalous dimensions of the extra matter fields. Actually it is a rather unclear matter what kinds of fixed point theories are possible. Here we simply assume that the following superpotential is realized at the infrared fixed point;

$$\begin{aligned} W &= \lambda D\bar{S}H_u + \bar{\lambda}\bar{D}SH_d \\ &\quad + \kappa(\bar{D}D)^2 + \kappa'(\bar{D}D)(\bar{S}S) + \kappa''(\bar{S}S)^2 \\ &\quad + M_T\bar{T}T + M_D\bar{D}D + M_S\bar{S}S + M_{S'}\bar{S}'S' + \mu H_u H_d, \end{aligned} \tag{20}$$

where the terms in the last line are supersymmetric mass terms. Then the Higgs fields have large positive anomalous dimensions due to the Yukawa couplings. In this case the anomalous dimensions are fixed to be $\gamma_{H_u} = \gamma_{H_d} = 1$ and also $\gamma_D = \gamma_{\bar{D}} = \gamma_S = \gamma_{\bar{S}} = -1/2$. It should be noted here that μ is suppressed [13], while the supersymmetric masses M_D and so on, which determine decoupling scale of the superconformal sector from the MSSM sector, are enhanced. The explicit relations may be written as

$$\mu(\mu_{\rm IR}) = \left(\frac{\mu_{\rm IR}}{\Lambda_{\rm SC}}\right)\mu(\Lambda_{\rm SC}), \tag{21}$$

[4] The running of the gauge couplings are affected slightly by large anomalous dimensions of the MSSM Higgs fields, though the energy scale where their couplings to the SC-sector become strong is asuumed to be rather low. Therefore gauge coupling unification is not manifestly maintained and needs some further considerations.

$$M_D(\mu_{\rm IR}) \;=\; \left(\frac{\mu_{\rm IR}}{\Lambda_{\rm SC}}\right)^{-1/2} M_D(\Lambda_{\rm SC}). \tag{22}$$

If the bare masses of the extra matter fields are given to be the same order as bare μ, the decoupling scale can be much higher than the electroweak scale. For example, the Giudice-Masiero mechanism [14] can induce the same order of supersymmetric masses, μ and $M_{T,D,S,S'}$ at the Planck scale, and furthermore, these are of the same order as those of SUSY breaking masses like gaugino and scalar masses.

It has been known that soft supersymmetry breaking parameters show novel renormalization properties, when the theory stays at the infrared attractive fixed point [15]-[21]. The gaugino mass and also the A-parameters are suppressed with some powers of the renormalization scale. The scalar masses enjoy sum rules at low energy irrespectively of their initial values. In the case that matter fields ϕ_i of a superconformal gauge theory carries gauge representation R_i and the superpotential contains non-vanishing interactions of $\phi_i\phi_j\phi_k$ and so on, the sum rules are found to be [18, 19, 21]

$$\sum_i T(R_i)m_i^2 \to 0, \qquad m_i^2+m_j^2+m_k^2 \to 0. \tag{23}$$

In the present model the sum rules show that soft masses of $D, \bar{D}, S, \bar{S}, H_u, H_d$ are suppressed. Here we used the left-right symmetry and assumed $m_D^2 = m_{\bar{D}}^2$, $m_S^2 = m_{\bar{S}}^2$. In this model soft masses not only of H_u but also of H_d are suppressed. The μB term is suppressed like μ, but the B parameter itself is not suppressed by the superconformal dynamics. Therefore the B-parameter must be small as an initial condition in order to make successful electroweak symmetry breaking.

It is possible to modify this model so that the soft mass of H_d is not suppressed. For this purpose let us introduce a pair of mirror Higgs H'_u and H'_d and assign odd matter parity to them. Also we change the matter parity of the MSSM singlet fields to $S:(+), \bar{S}:(-), S':(+), \bar{S}':(-)$. Then the superpotential allowed by this parity assignment is expected to be

$$\begin{aligned} W \;=\; & \lambda D\bar{S}H_u + \lambda'\bar{D}S'H'_u + \kappa(\bar{D}D)^2 + \kappa'(\bar{S}S')^2 \\ & + M_T\bar{T}T + M_D\bar{D}D + +\mu H_uH_d + \mu' H'_uH'_d. \end{aligned} \tag{24}$$

The Yukawa coupling with H_d is forbidden by the matter parity and therefore the soft scalar mass m_{H_d} is not suppressed. The mass M_D as well as M_T is enhanced and the superconformal sector decouples from the MSSM sector and enters confinement phase below this scale.

As well as the former model, the μ-parameter is suppressed. However the factor becomes milder and it is given at the decoupling scale as

$$\mu(M_D) = \left(\frac{M_D}{\Lambda_{\rm SC}}\right)^{1/2} \mu(\Lambda_{\rm SC}), \tag{25}$$

since only H_u has a large anomalous dimension. Explicitly we may suppose that the decoupling scale M_D is around a few TeV and $\Lambda_{\rm SC}$ is around $10 \sim 10^2$ TeV. Then the μ-parameter is suppressed about one order.

However what we are most concerned with is the MSSM radiative corrections to the low energy $m_{H_u}^2$ and their dependence on $m_{\tilde{t}}^2$. When the MSSM interactions are switched on, the soft scalar mass squared of H_u, $m_{H_u}^2$, is not simply suppressed. The relevant part of the RG flow equation for $m_{H_u}^2$ will be given by

$$\mu \frac{dm_{H_u}^2}{d\mu} = \Gamma_{H_u} m_{H_u}^2 + \frac{12}{16\pi^2} Y_t^2 m_{\tilde{t}}^2. \tag{26}$$

Here the factor Γ_{H_u} indicates the degree of power suppression of the scalar mass by the superconformal dynamics. (See Ref.[18, 20, 21].) The non-perturbative dynamics makes it difficult to know this factor, but it would be similar to the anomalous dimension γ_{H_u}. Let us evaluate $m_{H_u}^2$ at the decoupling scale M_D by using Eq. (26). If we ignore renormalization of Y_t and $m_{\tilde{t}}^2$, then it is immediately seen that the soft scalar mass behaves as[5]

$$m_{H_u}^2(M_D) \to -\frac{12}{16\pi^2 \Gamma_{H_u}} Y_t^2 m_{\tilde{t}}^2(M_D). \tag{27}$$

In practice, the top Yukawa coupling is also suppressed just like the μ-parameter. However it may be shown that $m_{H_u}^2(M_D)$ is given similarly even if taking account of the power correction to the Yukawa coupling. This equation should be compared with Eq. (7). Then it is noted that the large logarithmic factor disappears. This is because that the large radiative correction at high energy is erased by the attractive nature of superconformal dynamics, and the low energy Higgs mass is determined at the decoupling scale. Here we would stress again that the radiative correction to $m_{H_u}^2$ largely enhanced by the huge scale difference is the biggest factor causing the MSSM fine tuning problem. Therefore the degree of fine tuning with respect to $m_{\tilde{t}}^2$ (or gluino mass) can be remarkably improved by the superconformal dynamics. Also fine-tuning with respect to μ-parameter is dissolved simultaneously.

We may expect that large corrections to the Higgs mass and also to the Higgs quartic coupling are induced through decoupling of the superconformal sector, since H_u is coupled very strongly with this sector [6]. However note that the superconformal gauged matter fields coupled to H_u (D, S in the explicit model) undergo suppression of the scalar masses. According to the naive dimensional analysis [22], the correction to the Higgs soft mass due to decoupling is roughly evaluated as $\delta m_{H_u}^2 \sim m_{H_u}^2 m_D^2/M_D^2$, which is much smaller than $m_{H_u}^2$. Therefore

[5] Similar behavior has been studied in Ref.[18, 20, 21].

[6] The effective Higgs quartic coupling generated by decoupling of extra gauged matter fields has been discussed in Ref. [5]. However the soft supersymmetry breaking masses of the extra matter fields need to be large as 10 TeV order in order to ameliorate the fine tuning problem.

suppression of the soft mass of Higgs field to the electroweak scale is not destroyed by the decoupling effect of the superconformal sector. Similarly, however, the correction to the Higgs quartic coupling would be small in these models.

4 Generic aspects

In the previous section, we have shown concrete models leading to suppressed Higgs soft scalar masses and the suppressed μ-term, compared with other SUSY breaking masses. Here we collect results for the case that only the Higgs fields couple with generic superconformal sector, but MSSM matter fields do not couple. Then, we will discuss some phenomenological aspects of our scenario. The μ term is suppressed as

$$\mu(M_D) \approx \left(\frac{M_D}{\Lambda_{\rm SC}}\right)^{\gamma_{H_u}+\gamma_{H_d}} \mu(\Lambda_{\rm SC}), \tag{28}$$

where γ_{H_u} and γ_{H_d} are anomalous dimensions of $H_{u,d}$ induced by the superconformal sector. If H_d has negligible couplings, we have $\gamma_{H_d} \sim 0$. The soft scalar mass of H_u is also suppressed at M_D as

$$m^2_{H_u}(M_D) \sim -\frac{12}{16\pi^2 \Gamma_{H_u}} Y_t^2 m^2_{\tilde{t}}(M_D), \tag{29}$$

independently of initial values. The soft scalar mass of H_d is also suppressed when H_d couples with the superconformal sector [7]. When $Y_t^2/\Gamma_{H_u} \sim 1$ at M_D, we evaluate

$$m^2_{H_u}(M_D) \sim -0.08 \times m^2_{\tilde{t}}(M_D). \tag{30}$$

It is expected that $\mu(\Lambda_{\rm SC}) \sim m_{\tilde{t}}$ by a certain mechanism like the Giudice-Masiero mechanism. Then, the little hierarchy among $m^2_{H_u}$, μ^2 and other SUSY breaking terms like $m^2_{\tilde{t}}$ can be realized. For example, we would have $-m^2_{H_u}(M_D) \sim (100)^2$ $({\rm GeV})^2$ for $m_{\tilde{t}}(M_D) \sim 500$ GeV. Compared with Eq. (7), the fine-tuning parameter $\Delta_{m^2_{\tilde{t}}(M_D)}$ in our present scenario is reduced by the large logarithmic factor $\ln(\Lambda/v) \sim 30$. For example, we have $\Delta_{m^2_{\tilde{t}}(M_D)} \leq O(10)$ for $m_{\tilde{t}} \sim 500$ GeV. However, the decoupling scale M_D should not be far away from the weak scale.

In this section, we discuss other phenomenological implications of our spectrum. In the minimal supergravity model, most of parameter regions lead to a large value of μ compared with the bino mass. That implies that the lightest neutralino is purely bino-like, which is the lightest superpartner (LSP). In contrast, our scenario leads to a small value of μ. Thus, the lightest neutralino can be a mixture of the higgsino and bino including the case with the purely higgsino. Furthermore, the next lighter neutralino and the lightest chargino can

[7] The RG flow of the top Yukawa coupling is the similar to one of μ. We need $Y_t(v) \sim 1$ to realize the top mass. That implies that $Y_t(\Lambda_{\rm SC})$ must be large, depending how much hierarchy we expect between μ and other SUSY breaking masses e.g. $m_{\tilde{t}}$.

have similar masses. This brings a drastic change in the relic abundance of the LSP [23]. However the mass spectrum depends on the detail of models, specially on the gaugino masses. Explicit considerations on the relic abundance is beyond our scope.

Also the situation on the charge and/or color breaking and unbounded-from-below constraints can change. The most serious unbounded-from-below direction of the MSSM scalar potential involves the Higgs and slepton fields $\{H_u, \tilde{\nu}_{L_i}, \tilde{e}_{L_j}, \tilde{e}_{R_j}\}$, i.e. the so-called UFB-3 direction [24, 25]. We use the stationary conditions $\partial V/\partial\phi = 0$ for $\phi = \{\tilde{\nu}_{L_i}, \tilde{e}_{L_j}, \tilde{e}_{R_j}\}$, and write the field values of $\{\tilde{\nu}_{L_i}, \tilde{e}_{L_j}, \tilde{e}_{R_j}\}$ in terms of H_u. Then, the potential along the UFB-3 direction is written as

$$V_{\mathrm{UBF-3}} = (m^2_{H_u} + m^2_{\tilde{L}_i})|H_u|^2 + \frac{|\mu|}{Y_{e_j}}(m^2_{\tilde{L}_j} + m^2_{\tilde{e}_j} + m^2_{\tilde{L}_i})|H_u| - \frac{2m^4_{\tilde{L}_i}}{g^2 + g'^2}, \tag{31}$$

if the following inequality is satisfied,

$$|H_u| > \sqrt{\frac{\mu^2}{4Y^2_{e_j}} + \frac{4m^2_{\tilde{L}_i}}{g^2 + g'^2}} - \frac{|\mu|}{2Y_{e_j}}, \tag{32}$$

where Y_{e_j} is the lepton sector Yukawa coupling. Otherwise,

$$V_{\mathrm{UBF-3}} = m^2_{H_u}|H_u|^2 + \frac{|\mu|}{Y_{e_j}}(m^2_{\tilde{L}_j} + m^2_{\tilde{e}_j})|H_u| + \frac{1}{8}(g^2 + g'^2)\left[|H_u|^2 + \frac{|\mu|}{Y_{e_j}}|H_u|\right]^2. \tag{33}$$

The Higgs soft mass squared $m^2_{H_u}$ is strongly driven negatively by the stop mass, that is, the gluino mass, while radiative corrections to $m^2_{\tilde{L}_j}$ due to the wino mass is small. Therefore the quantity $m^2_{H_u} + m^2_{\tilde{L}_i}$ tends to be negative. Thus, the large negative contribution due to $m^2_{H_u}$ is serious along the UFB-3 direction, if the initial condition of $m^2_{H_u}$ and $m^2_{\tilde{L}_j}$ are of the same order like the minimal supergravity model [25]. In our scenario, we have the spectrum $|m^2_{H_u}| < m^2_{\tilde{L}_j}$ naturally. Therefore the UFB-3 bound can be relaxed also.

5 Conclusion and discussions

The experimental lower bound of the lightest Higgs boson leads that the mass parameters in the Higgs sector of the MSSM must be fine-tuned to a few %-order. There have been various modifications to avoid this problem so far. In the most cases, additional contributions for the quartic couplings of the Higgs bosons are introduced and the relatively light soft scalar masses are also assumed.

Here we have studied the scenario that only the μ term and SUSY breaking mass terms in the Higgs sector are suppressed compared with other SUSY breaking masses, e.g. the stop mass. Our scenario can be realized by the superconformal dynamics, which can alter the RG running for soft scalar mass

of the Higgs fields at low energy. The large radiative correction to the Higgs mass generated at higher energy can be erased. Its decoupling scale must be a few TeV or less. Our illustrating models can naturally generate such hierarchy, $\mu^2, m_{H_u}^2 \ll m_{\tilde{t}}^2 \ll M_D^2$, in the case that all of SUSY breaking masses have the same order of initial values and the μ term and supersymmetric mass terms in the superconformal sector are induced, e.g. by the Giudice-Masiero mechanism. Thus the mass spectrum in our scenario can relieve the fine-tuning problem. In general threshold effects due to the decoupling may induce additional quartic couplings of the Higgs fields and affect the lightest Higgs mass. These effects altogether work to relieve the fine-tuning problem, although the decoupling effects seem to be small in the models explicitly given here.

As other phenomenological aspects, small values of μ^2 and $m_{H_u}^2$ are significant. The smallness of μ affects on mixing of gaugino and higgsino in the neutralino and chargino states. That would drastically change the relic abundance of the LSP. Furthermore, compared with slepton masses the smallness of $|m_{H_u}^2|$ is favorable from the viewpoint of the most serious unbounded-from-below direction.

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